EFFICIENT PATTERN MATCHING IN ELASTIC-DEGENERATE TEXTS

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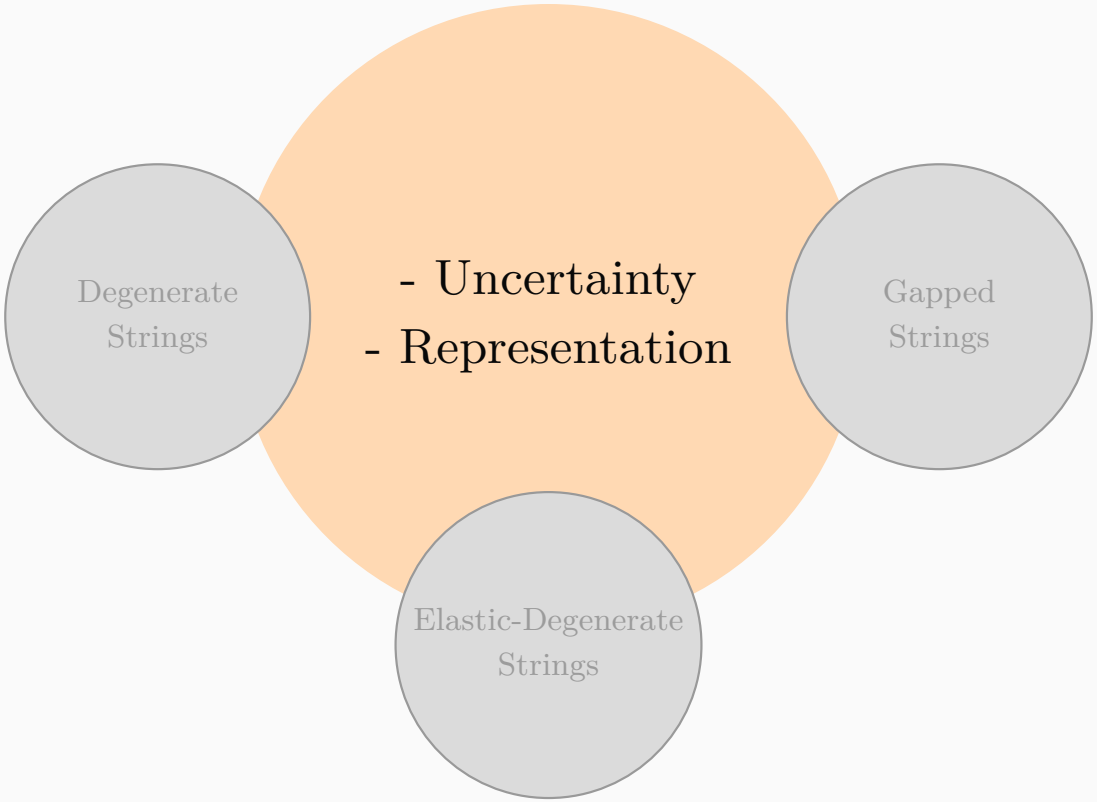
Outlines

INTRODUCTION

Introduction



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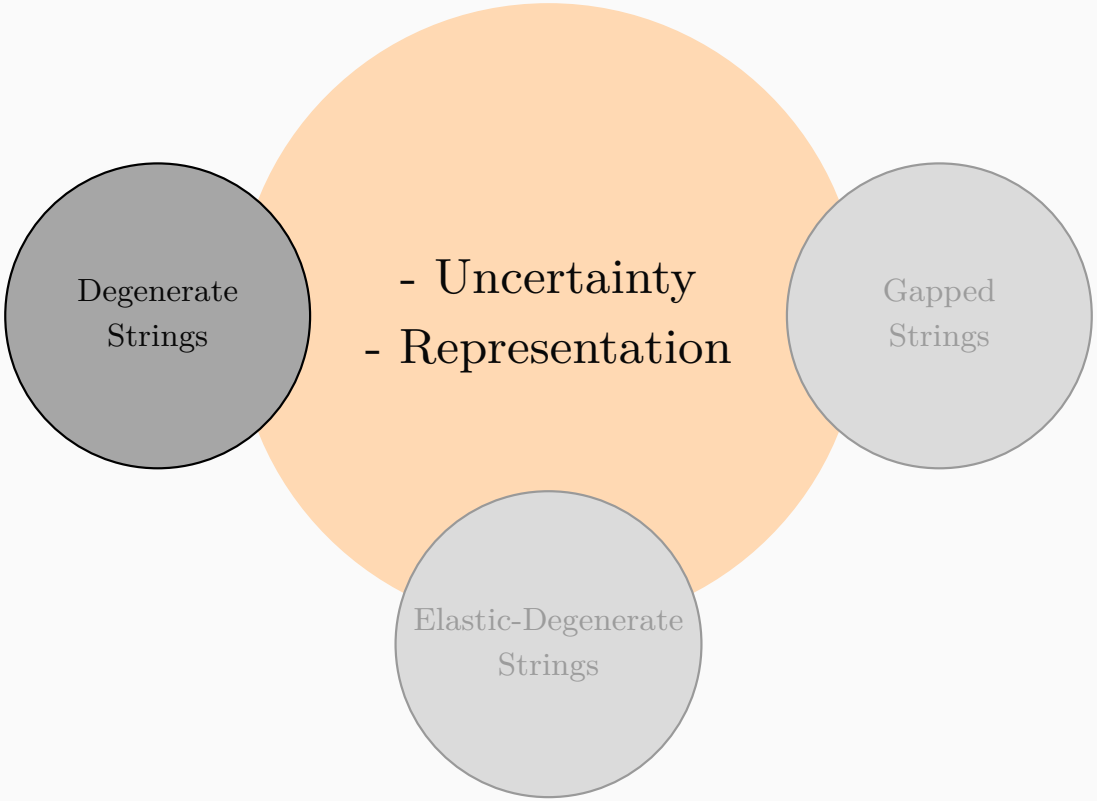

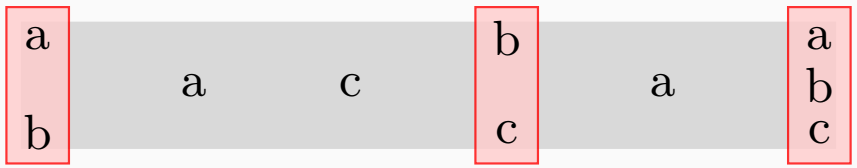

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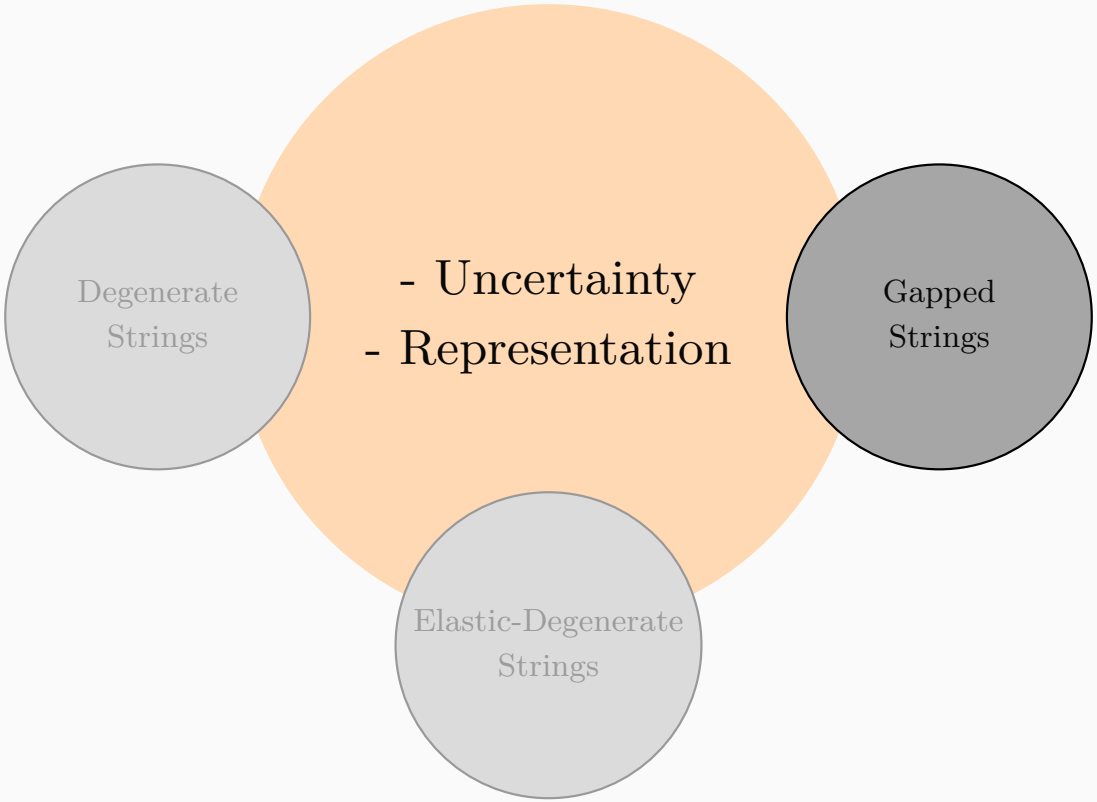
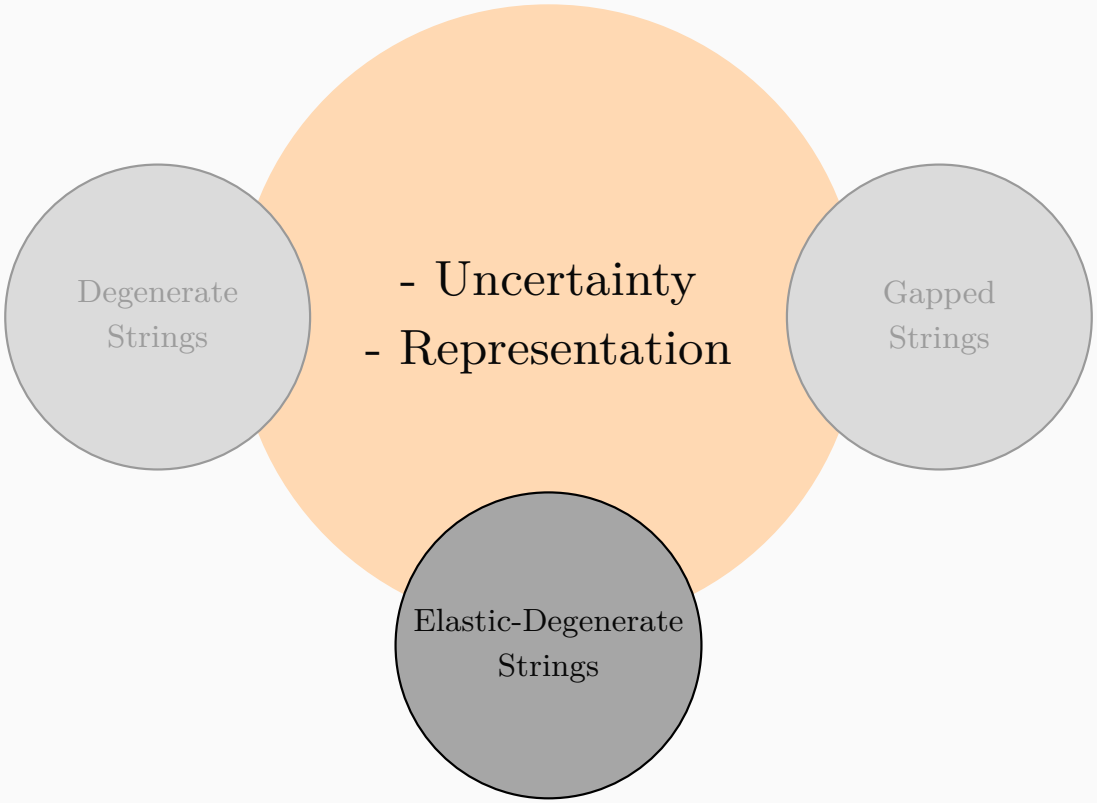
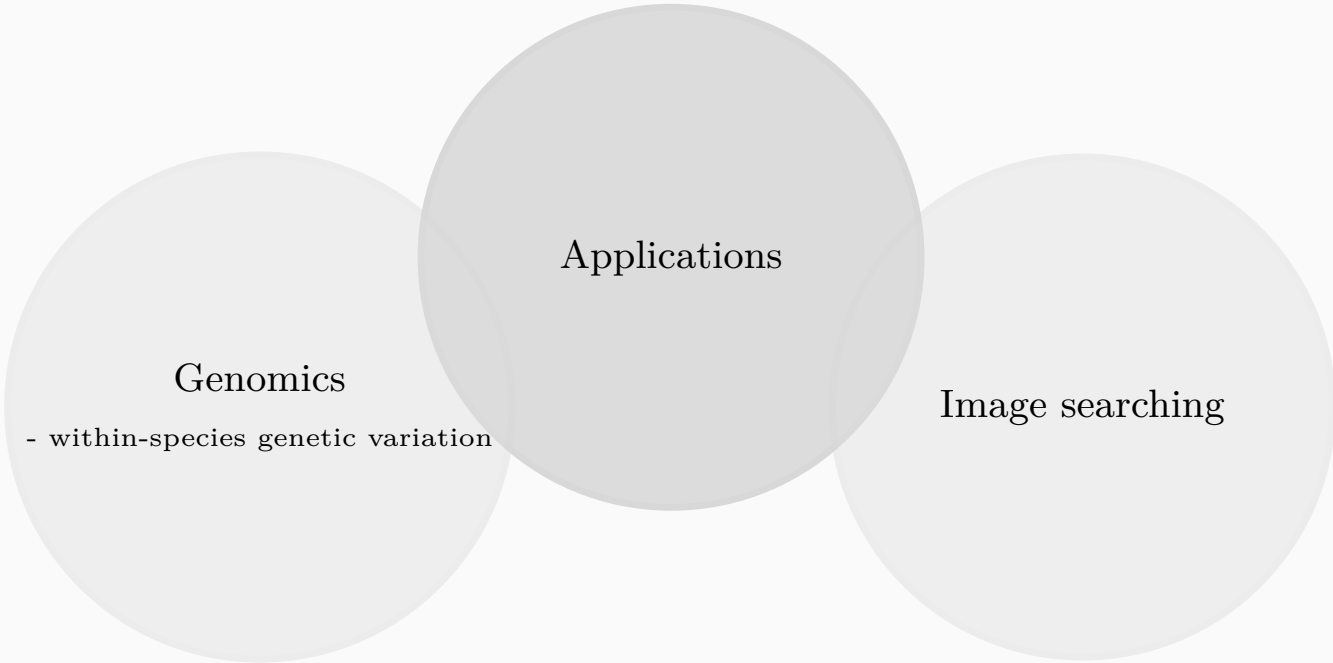


$a\ b\ b\ c$ | $2 - -4$ | $c\ c\ a$ | $3 - -6$ | $b\ b$

Introduction



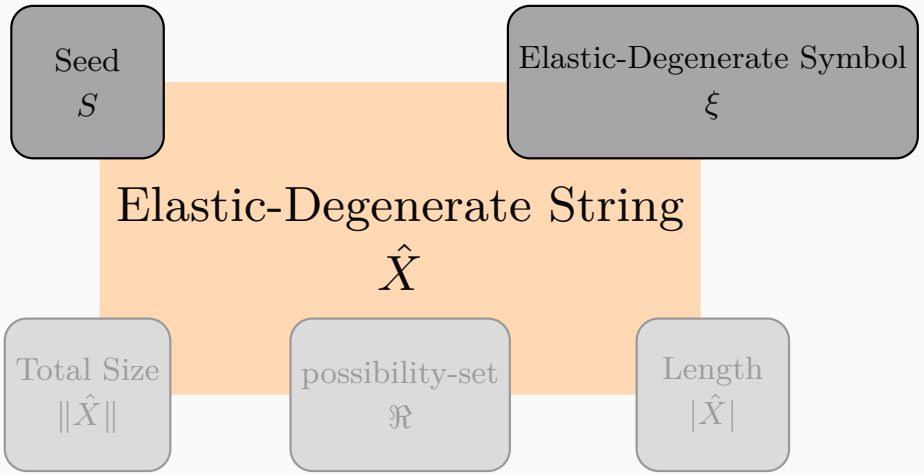
Motivation


Applications
Genomics
- within-species genetic variation
Image searching

TECHNICAL BACKGROUND

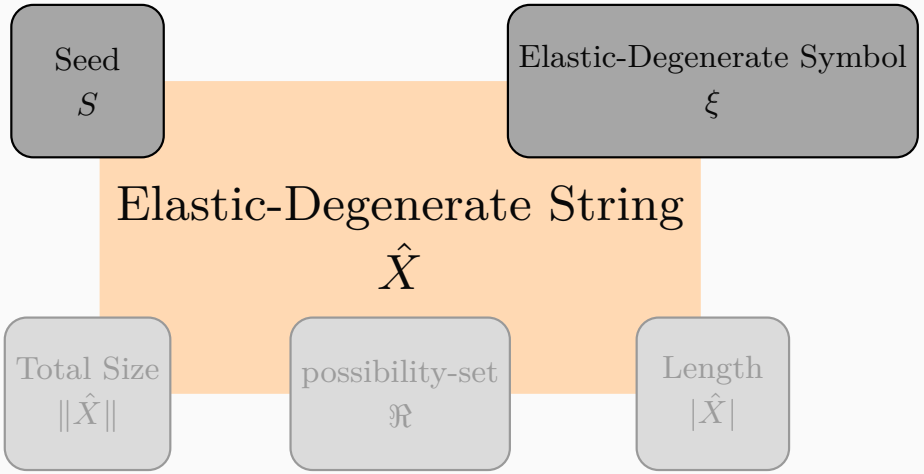
Terminology I

$$\hat{X} = S_1\xi_1 S_2\xi_2 S_3 \ldots S_{k-1}\xi_{k-1}S_k$$



Terminology I

$$\hat{X} = S_1 \begin{bmatrix} E_{1,1} \\ E_{1,2} \\ \vdots \\ E_{1,|\xi_1|} \end{bmatrix} S_2 \begin{bmatrix} E_{2,1} \\ E_{2,2} \\ \vdots \\ E_{2,|\xi_2|} \end{bmatrix} S_3 \ldots S_{k-1} \begin{bmatrix} E_{k-1,1} \\ E_{k-1,2} \\ \vdots \\ E_{k-1,|\xi_{k-1}|} \end{bmatrix} S_k$$



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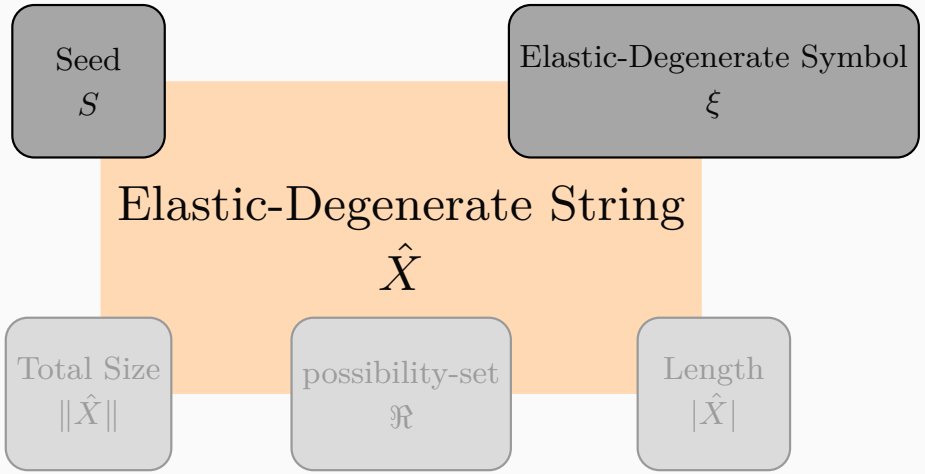
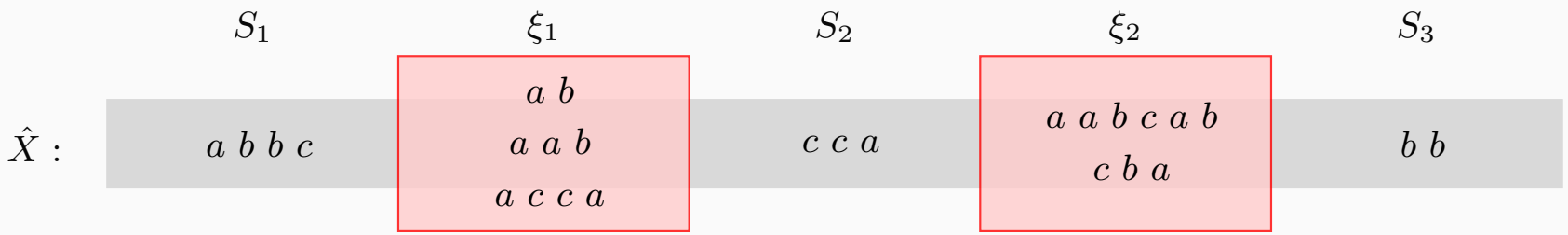


Example



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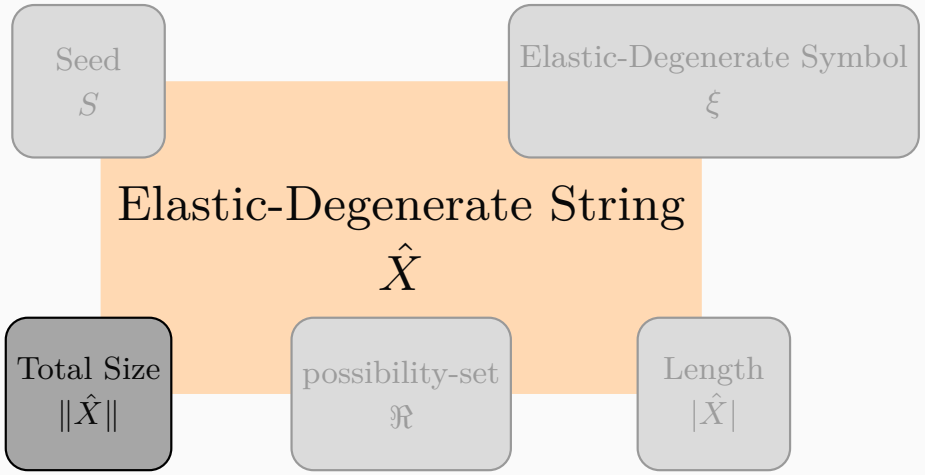
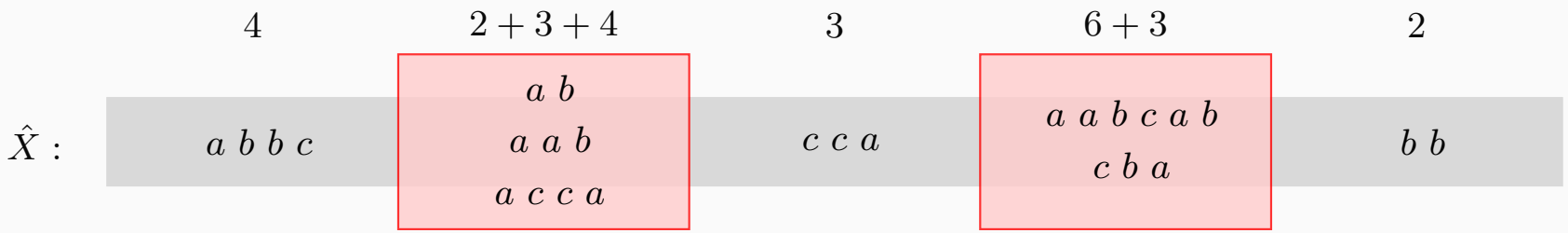


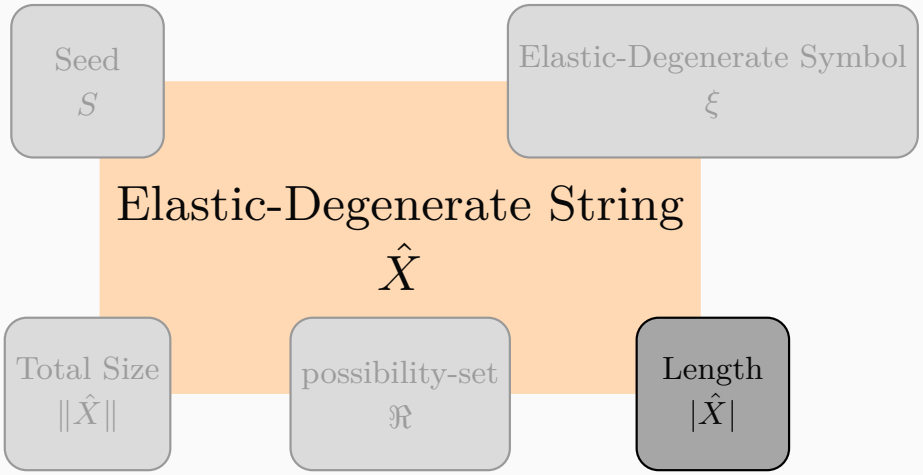
$$\|\hat{X}\| = \sum_{i=1}^{k} |S_i| + \sum_{i=1}^{k-1} \sum_{j=1}^{|\xi_i|} |E_{i,j}|$$

Example



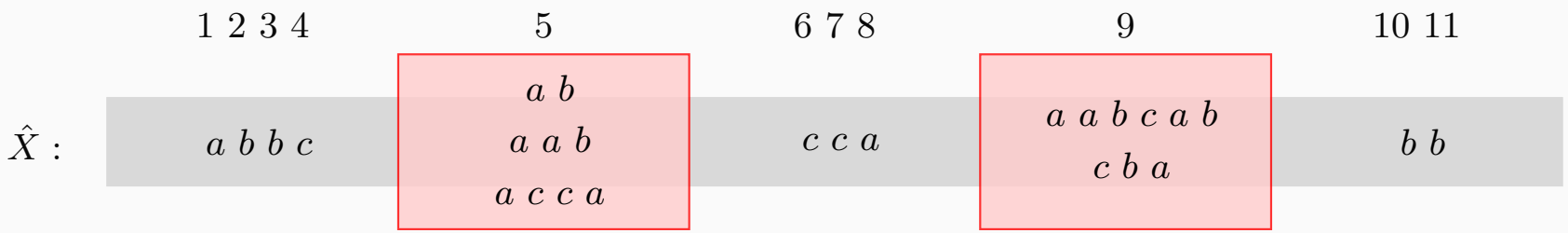
Total size ($\|\hat{X}\|$) = 27

Terminology I



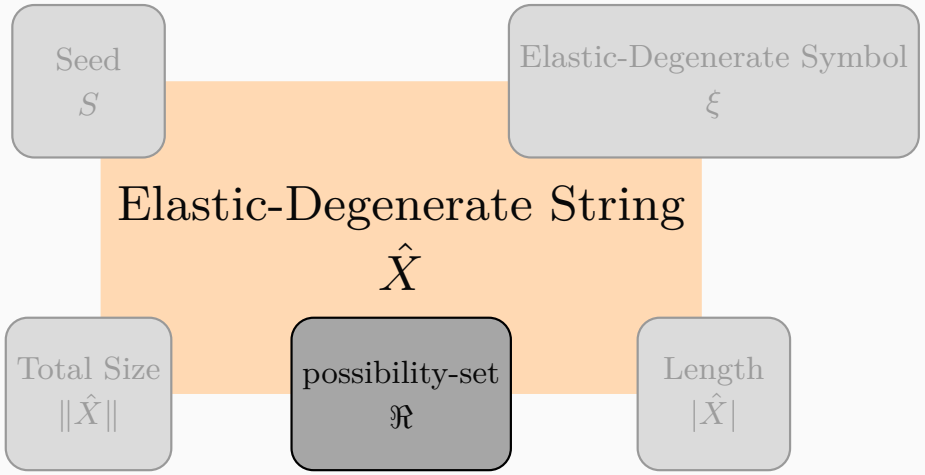
$$|\hat{X}| = \sum_{i=1}^{k} |S_i| + k - 1$$

Example



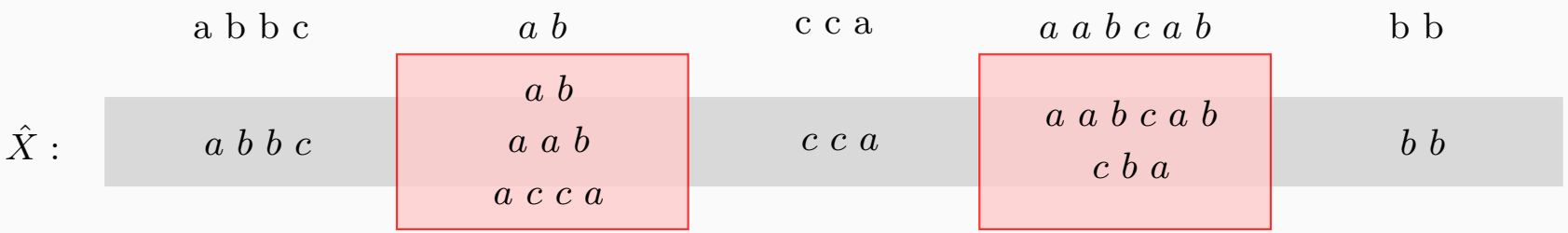
Length $(|\hat{X}|) = 11$

Terminology I



$\{S_1 E_{1,r_1} S_2 E_{2,r_2} \dots E_{k-1,r_{k-1}} S_k\} \quad \forall r_i, 1 \leq i \leq k-1 \text{ such that } 1 \leq r_i \leq |\xi_i|$

Example



$\Re = \{abbcabccaaabcabbb, abbcabccacbabb, abbcaabccaaabcabbb, abbaabccacbabb, abbcaccaccaaabcabbb, abbcaccaccacbabb\}$

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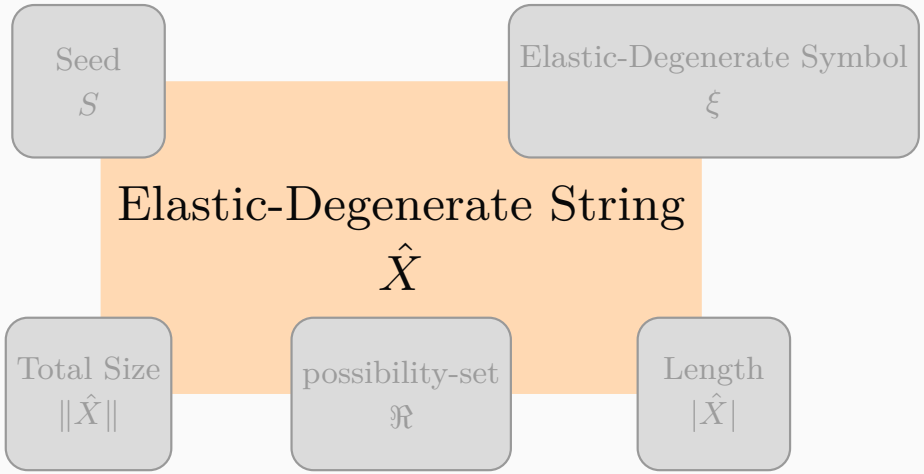

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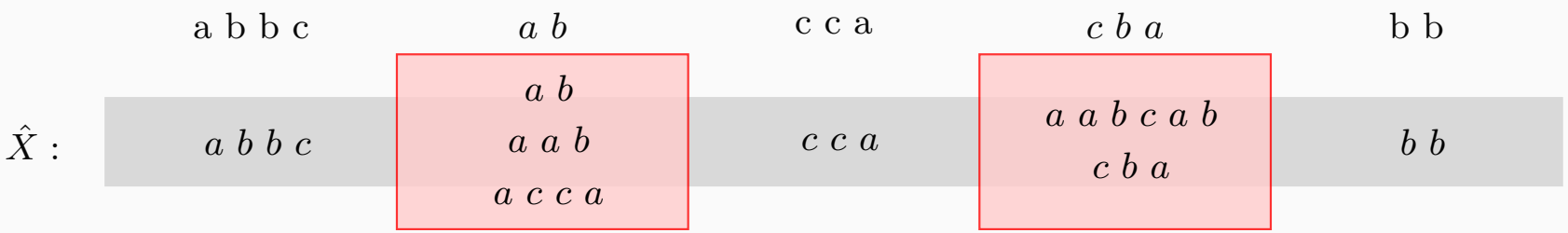

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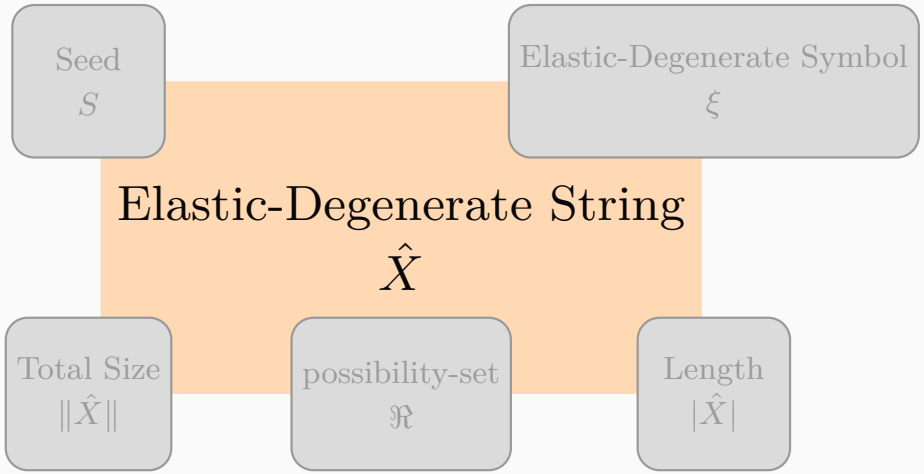

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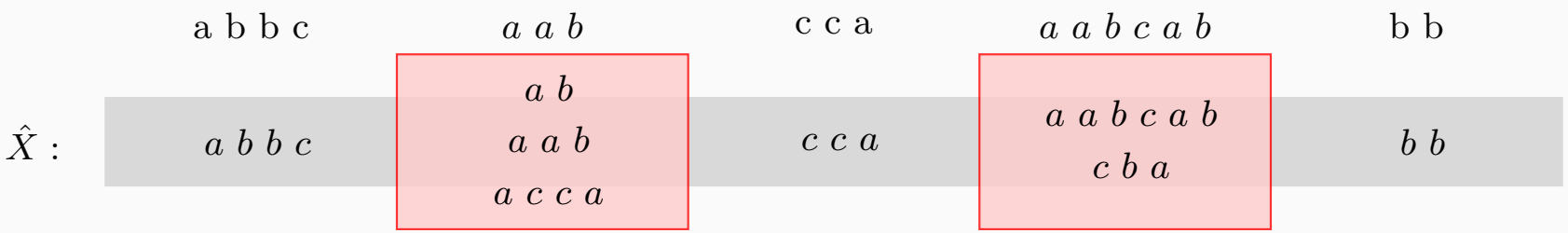

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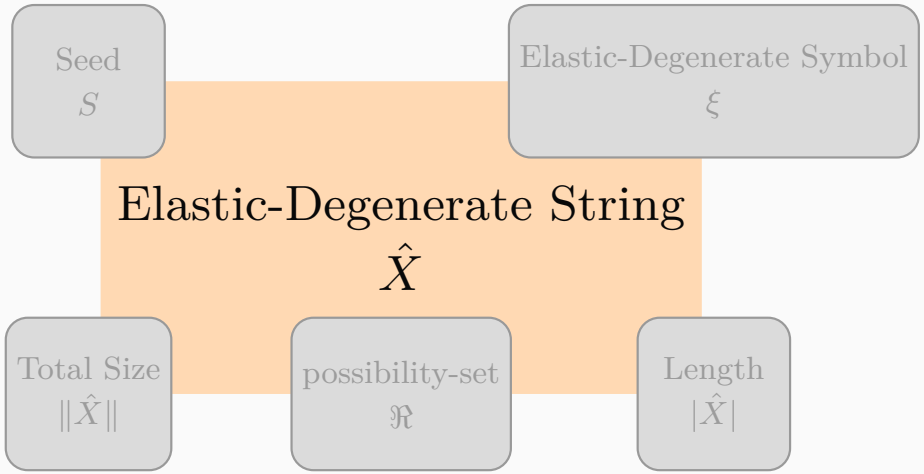

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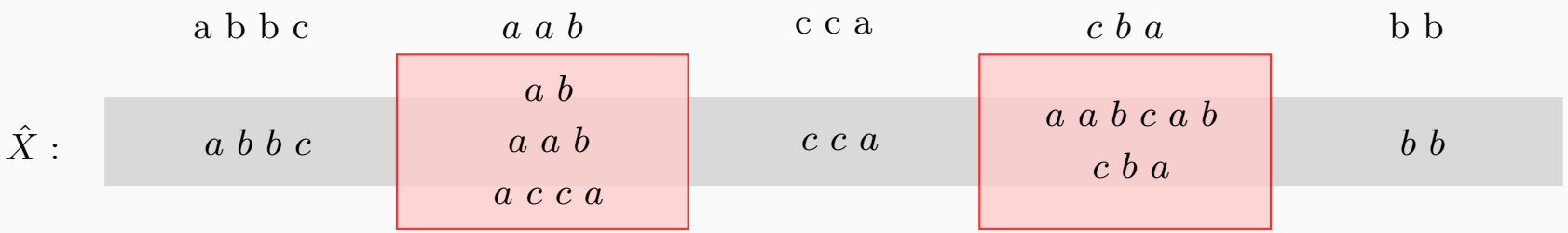

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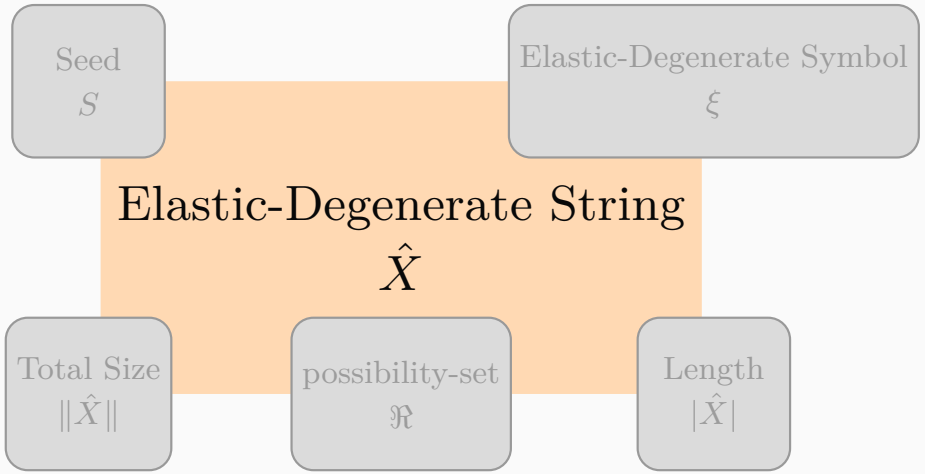

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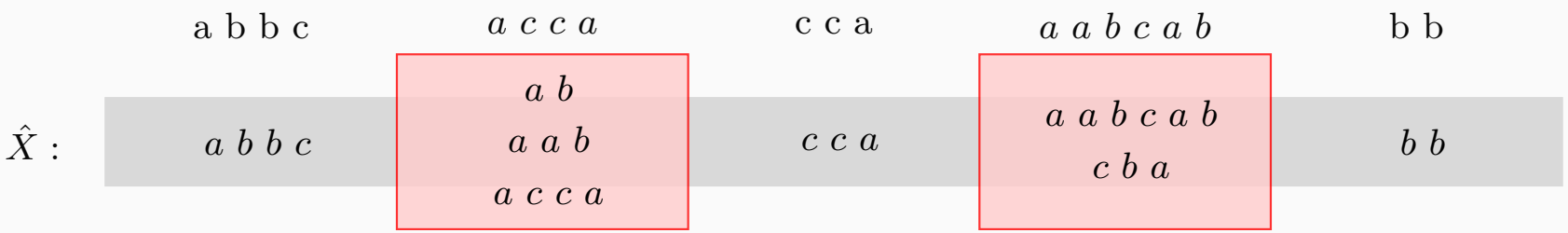

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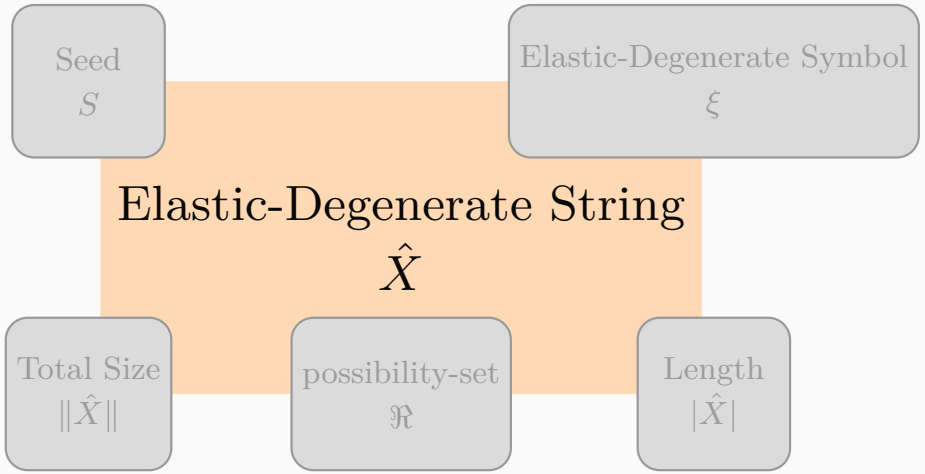

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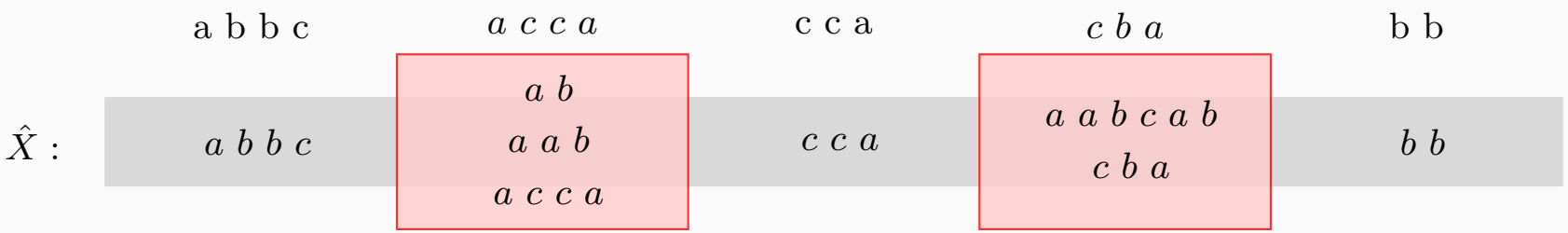

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Matching ($\hat{X} \simeq Y$)

An elastic-degenerate string $\hat{X}$ with k seeds and a solid string Y are said to match, denoted by $\hat{X} \simeq Y$, if, and only if, there exists a solid string $S = S_1 E_{1,r_1} S_2 E_{2,r_2} \cdots E_{k-1,r_{k-1}} S_k, 1 \leq r_i \leq |\xi_i|$, obtained from $\hat{X}$ (i.e. $S \in \Re$ of $\hat{X}$), such that $S = UYV$, where $U, V \in \Sigma^*$, satisfying:

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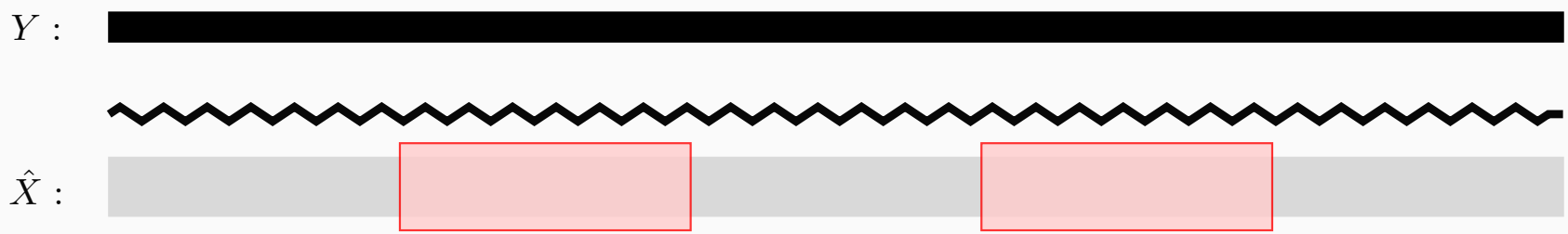
Illustration



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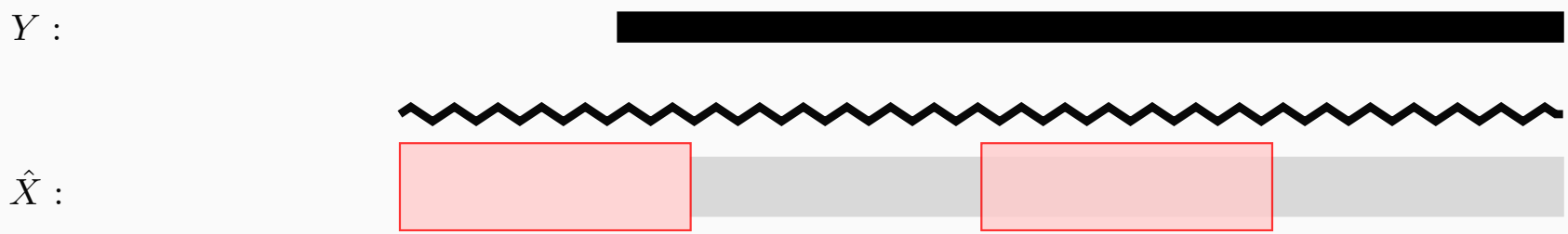

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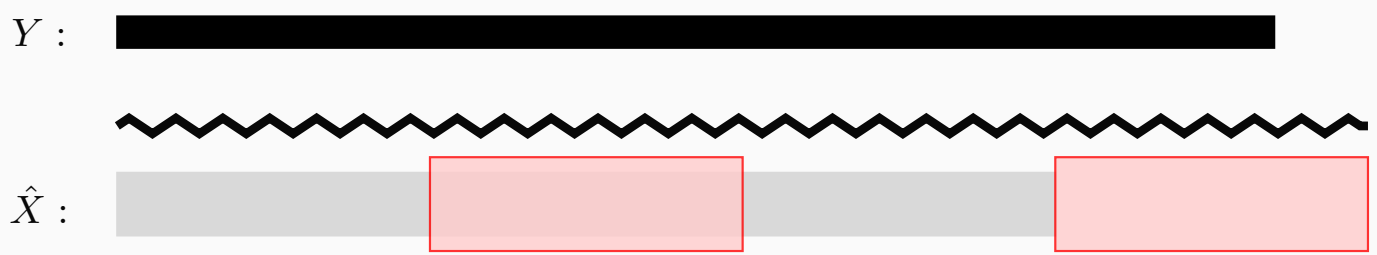

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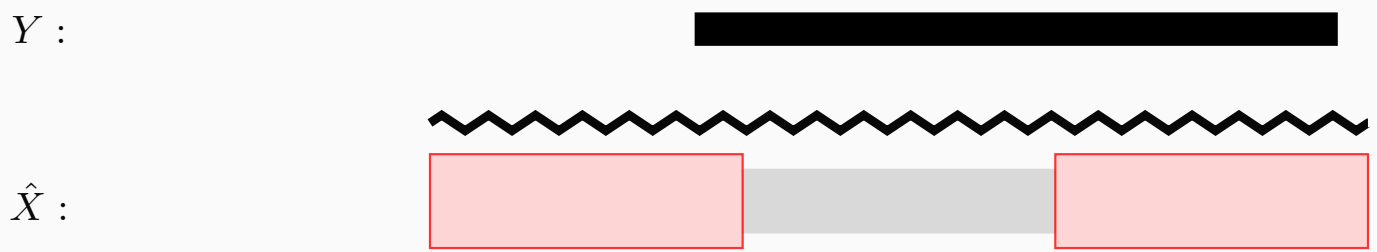
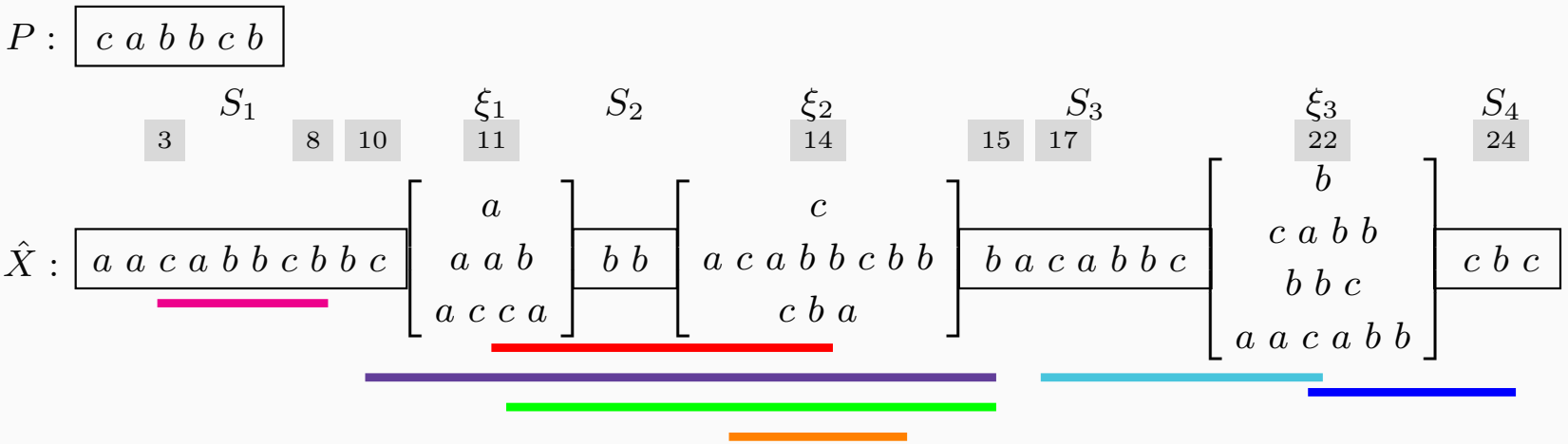


Occurrence ($\hat{X} \simeq Y$)

In an elastic-degenerate string (text) $\hat{T}$, a solid string (pattern) P is said to have an occurrence starting and ending at positions i and j respectively, if $P \simeq \hat{T}[i\,..\,j]$. An occurrence is represented as the pair of starting position i (head) and ending position j (tail).

Illustration



Occurrences: $(3, 8), (10, 15), (11, 14), (11, 15), (14, 14), (17, 22), (22, 24)$

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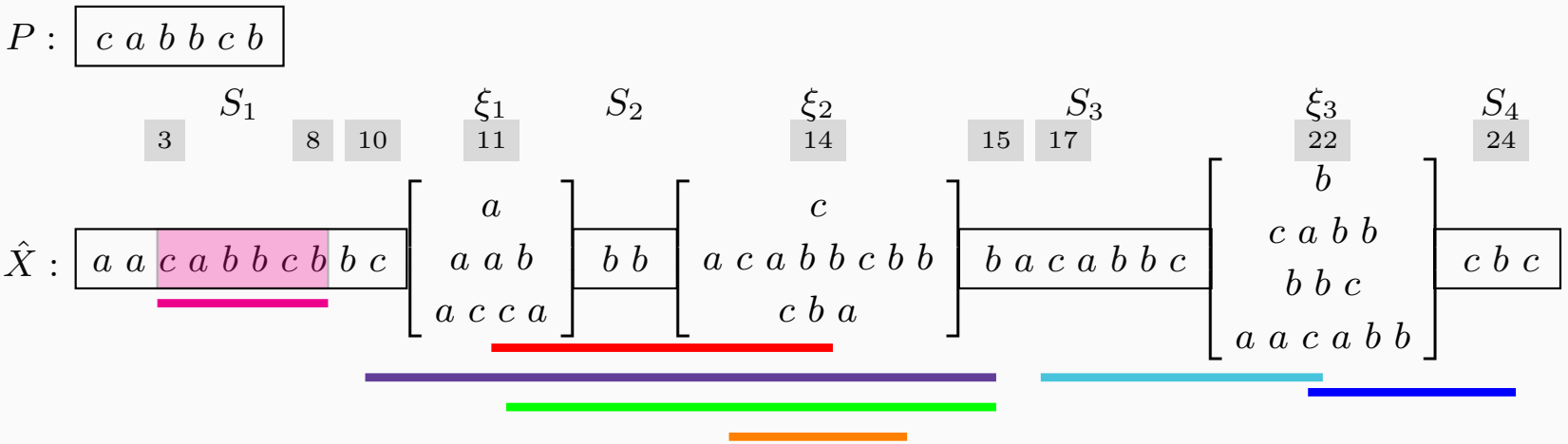

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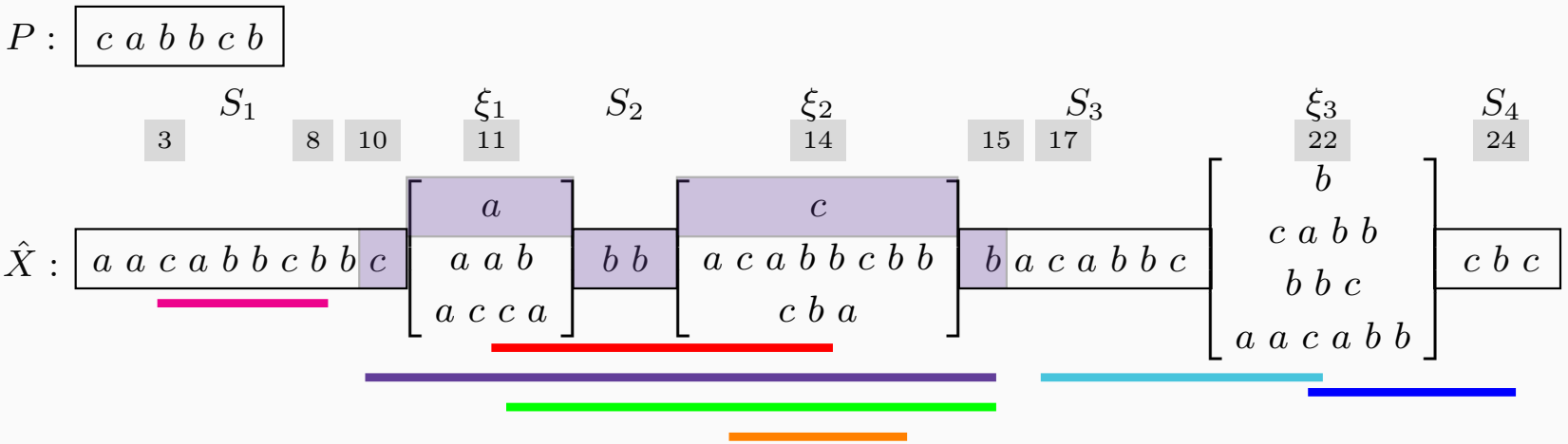

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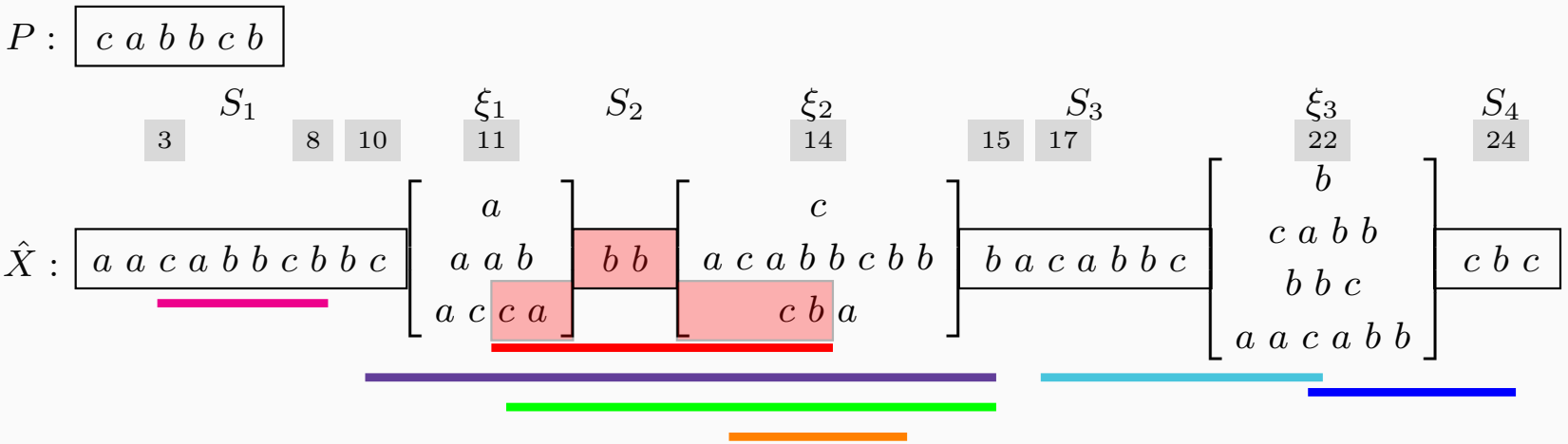

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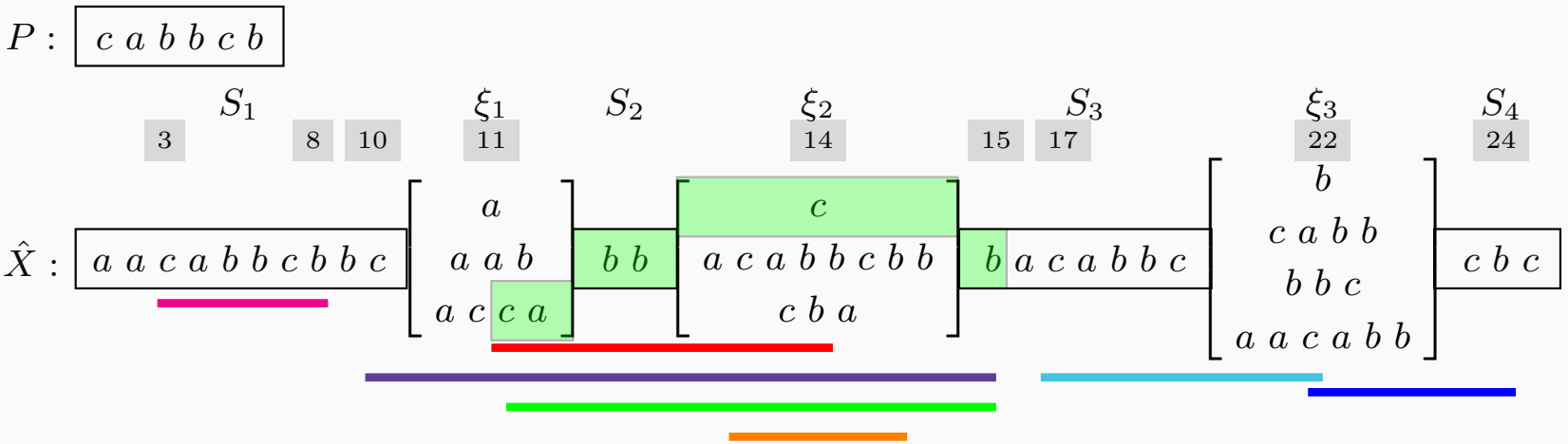

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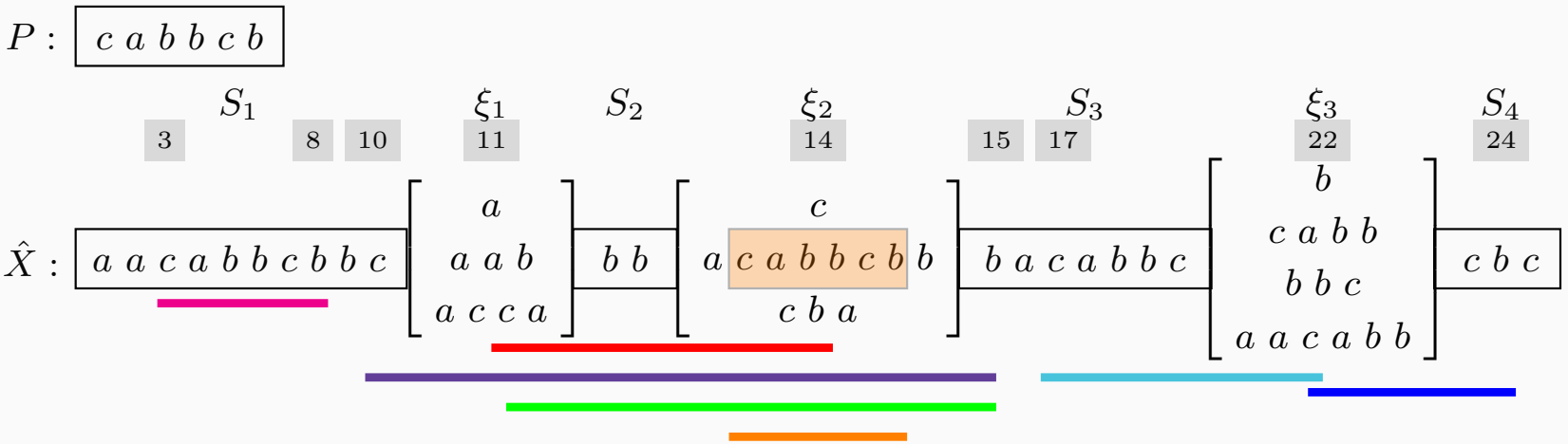

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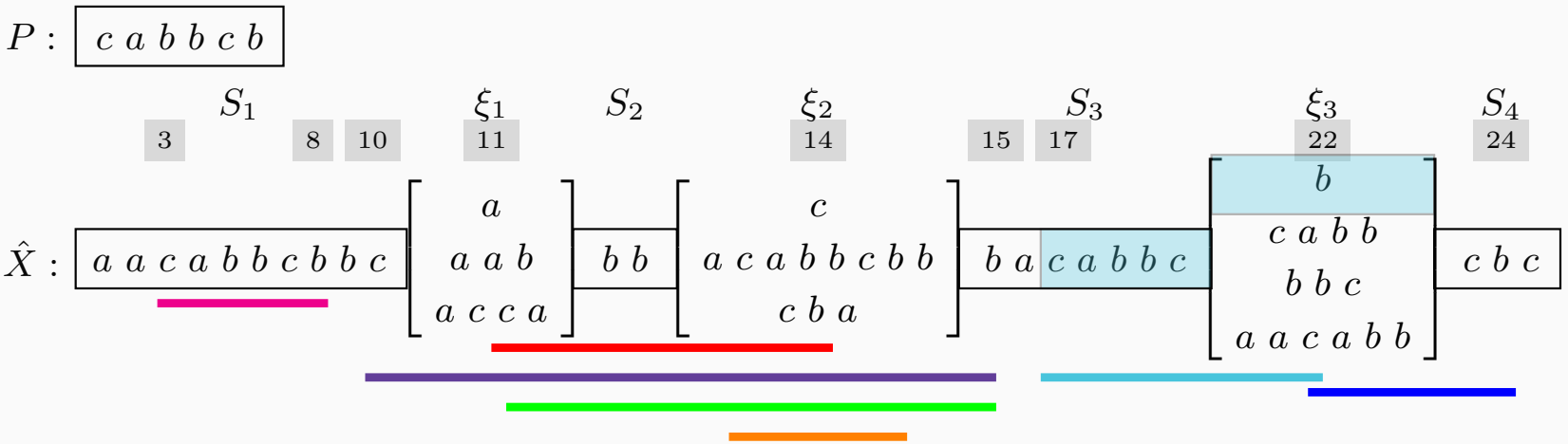
In an elastic-degenerate string (text) $\hat{T}$, a solid string (pattern) P is said to have an occurrence starting and ending at positions i and j respectively, if $P \simeq \hat{T}[i\,..\,j]$. An occurrence is represented as the pair of starting position i (head) and ending position j (tail).

Illustration



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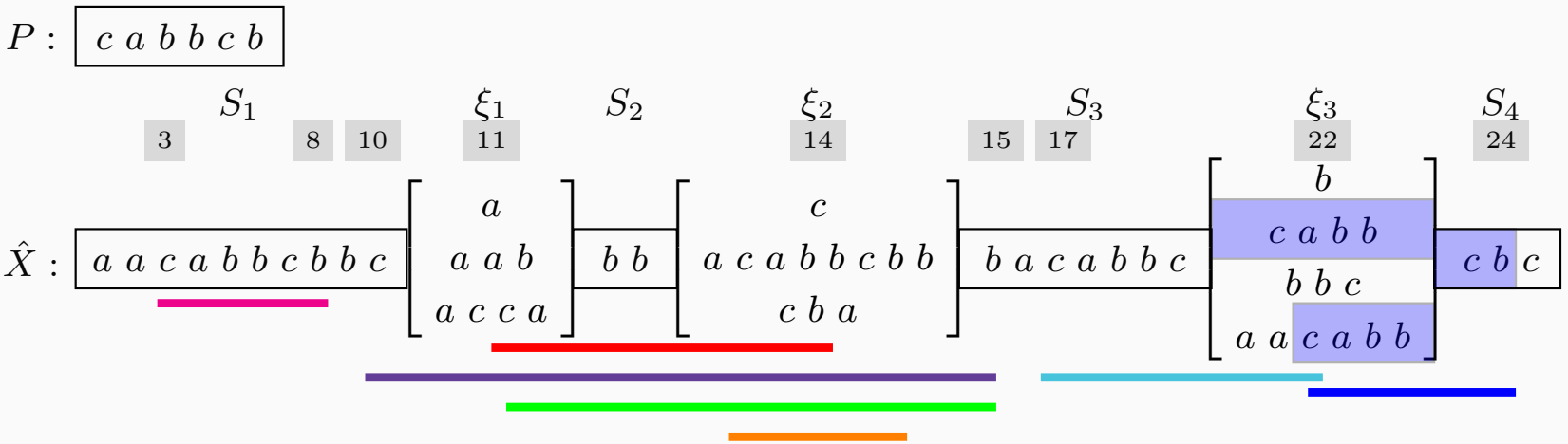

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Suffix Tree ([Crochemore et al., 2007])

- The suffix tree $\mathcal{S}(X)$ of a non-empty string X of length n is a compact trie representing all the suffixes of X such that $\mathcal{S}(X)$ has n leaves, labelled from 1 to n.
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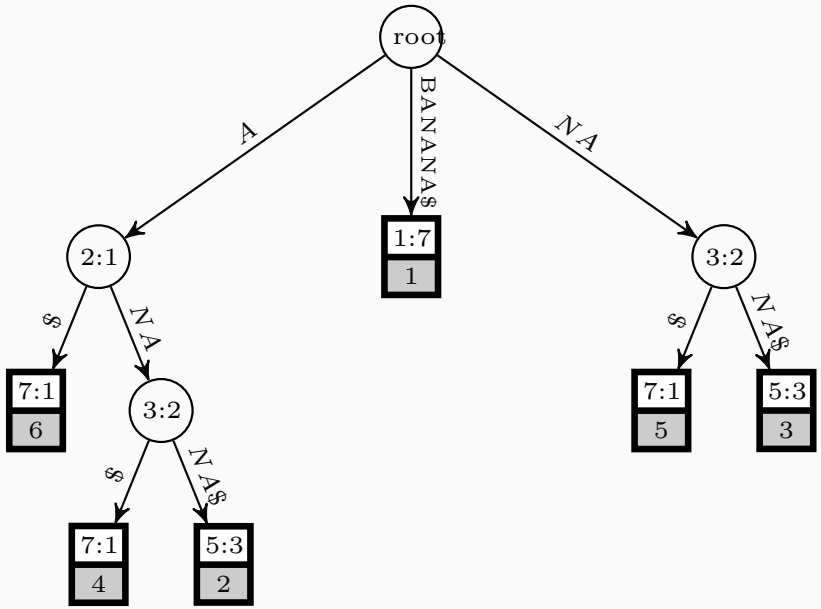

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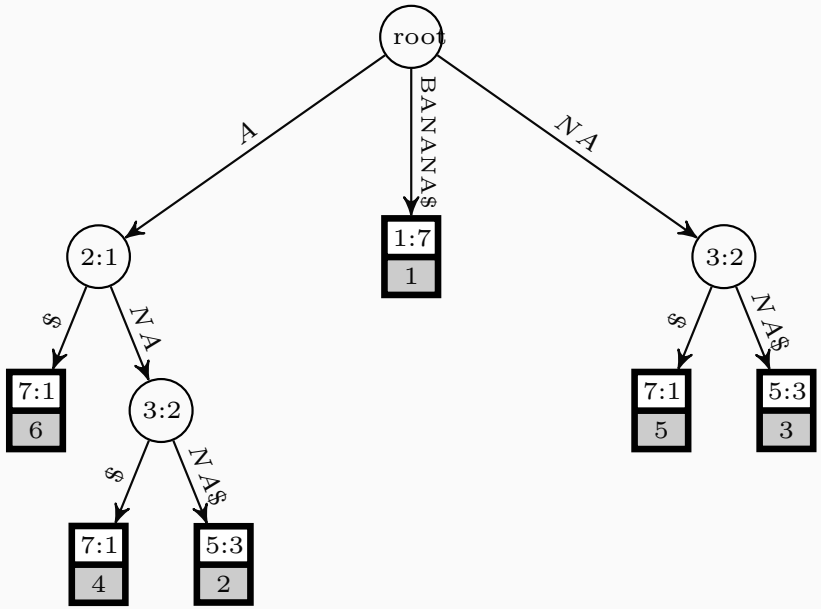

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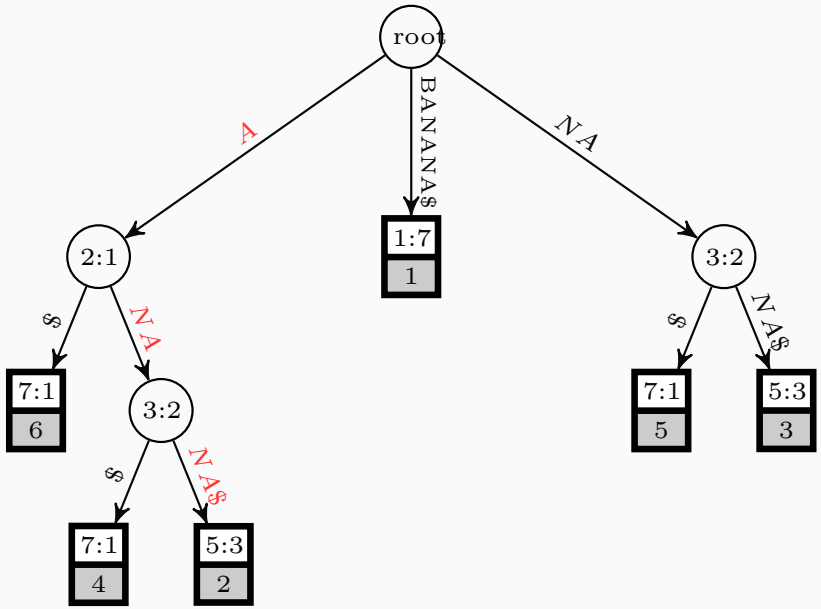

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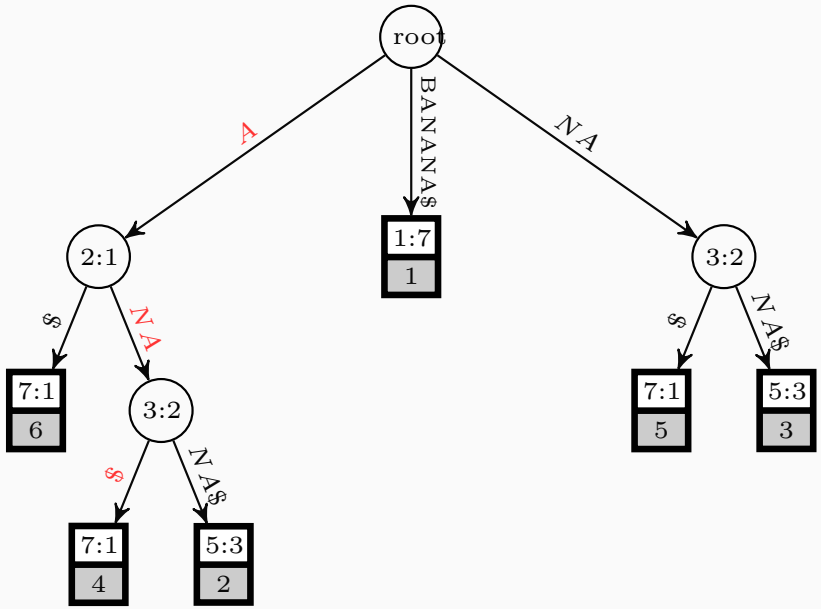

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KMP Algorithm ([Knuth et al., 1977])

- A linear time algorithm for finding all occurrences of a pattern P in a text T.
- It pre-processes P by computing a failure function f that indicates the maximum possible shift using previously performed symbol comparisons.
- By using the failure function, it achieves an optimal search time of $\mathcal{O}(n)$ after $\mathcal{O}(m)$-time pre-processing, where n is the length of T and $m < n$ is the length of P.

ALGORITHM

Problem Definition

Input

An elastic-degenerate text $\hat{T} = S_1\xi_1 S_2 \dots \xi_{k-1} S_k$ of length n and total size N, a pattern P of length $m < N$.

Output

All the occurrences of P in $\hat{T}$.

Types of occurrences

- Type 1: Solid starting position
- Type 2: Elastic-Degenerate starting position

Algorithm

- Preprocessing
- Search
  - Type 1:
  - Type 2:

Algorithm

- Preprocessing
  - Compute failure-function of P.
  - Create the generalised suffix tree $\mathbb{S}_S$ for the set $\{P, S_1, S_2, \ldots, S_k\}$ of strings.
  - Create the generalised suffix tree $\mathbb{S}_\xi$ for the set $\{P\} \cup \xi_1 \cup \xi_2 \cup \ldots \cup \xi_{k-1}$.
  - Pre-process these two suffix trees so as to answer LCA queries in constant time.
- Search
  - Type 1:
  - Type 2:

Algorithm

- Preprocessing
- Search
  - Type 1:
    - Use KMP to search (shifting using failure function) in some seed S_i.
    - When an elastic-degenerate symbol is hit, mark the corresponding position: Incremental extension of marked positions using LCP queries.
  - Type 2:

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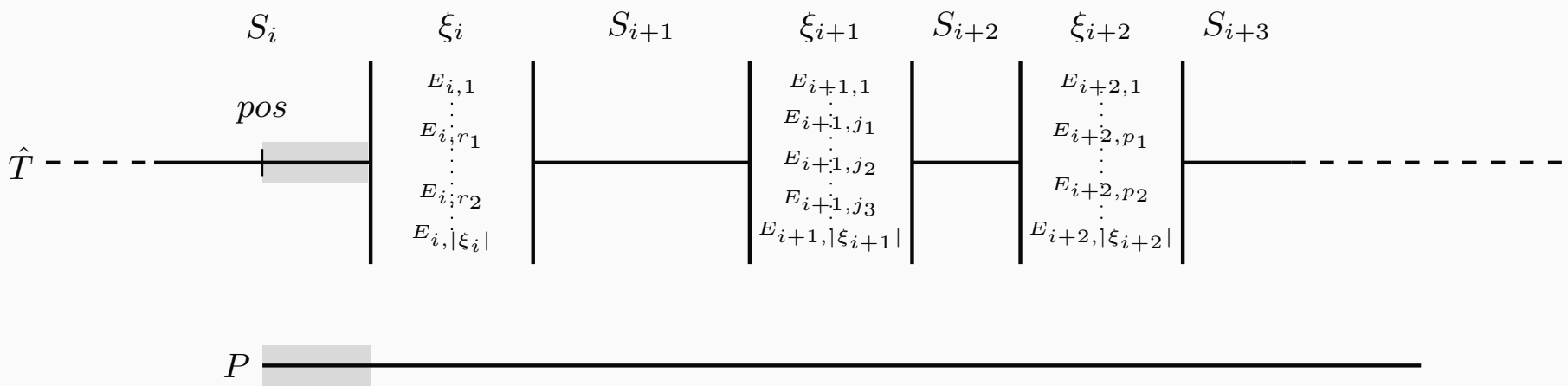

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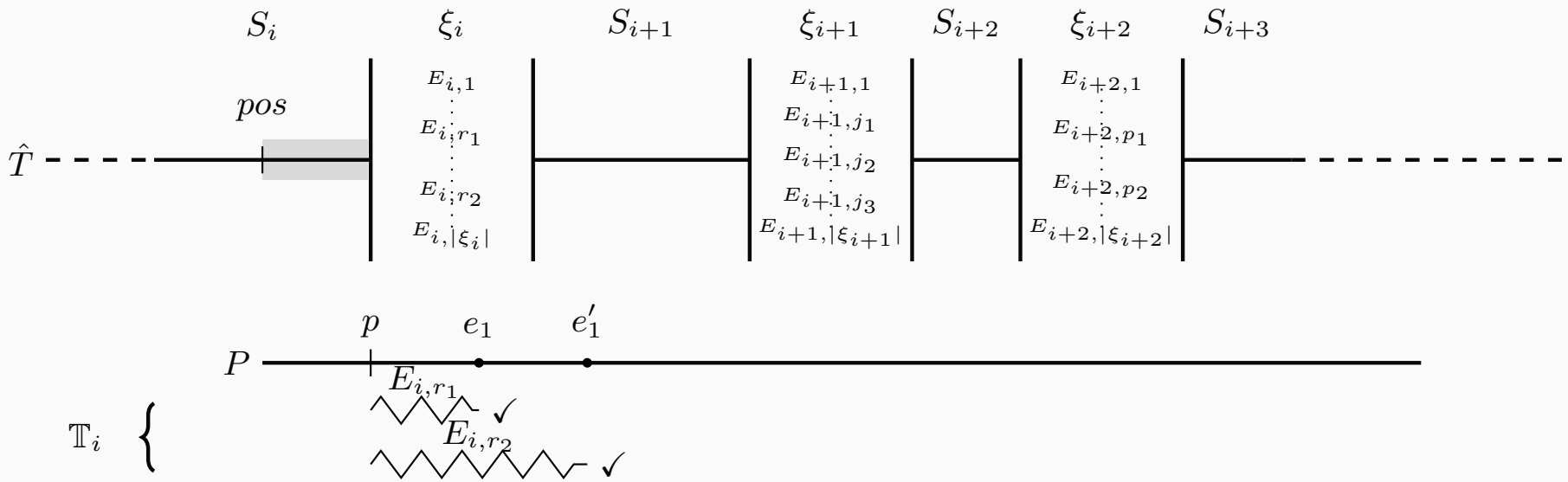

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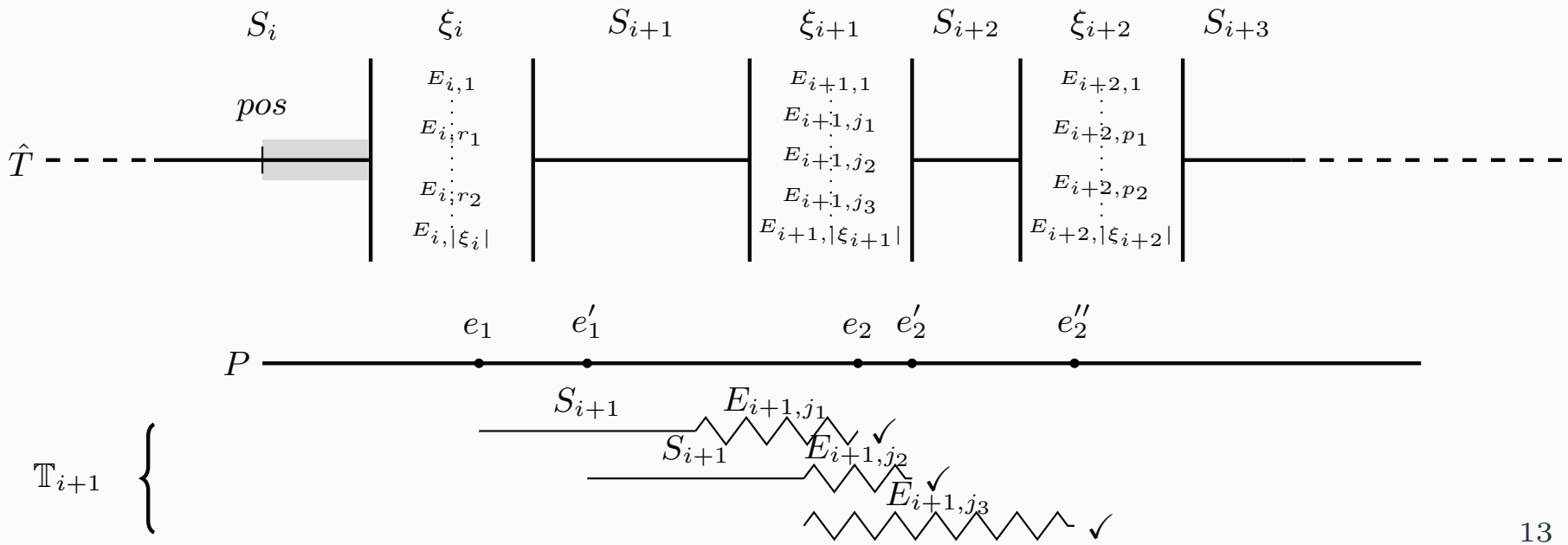

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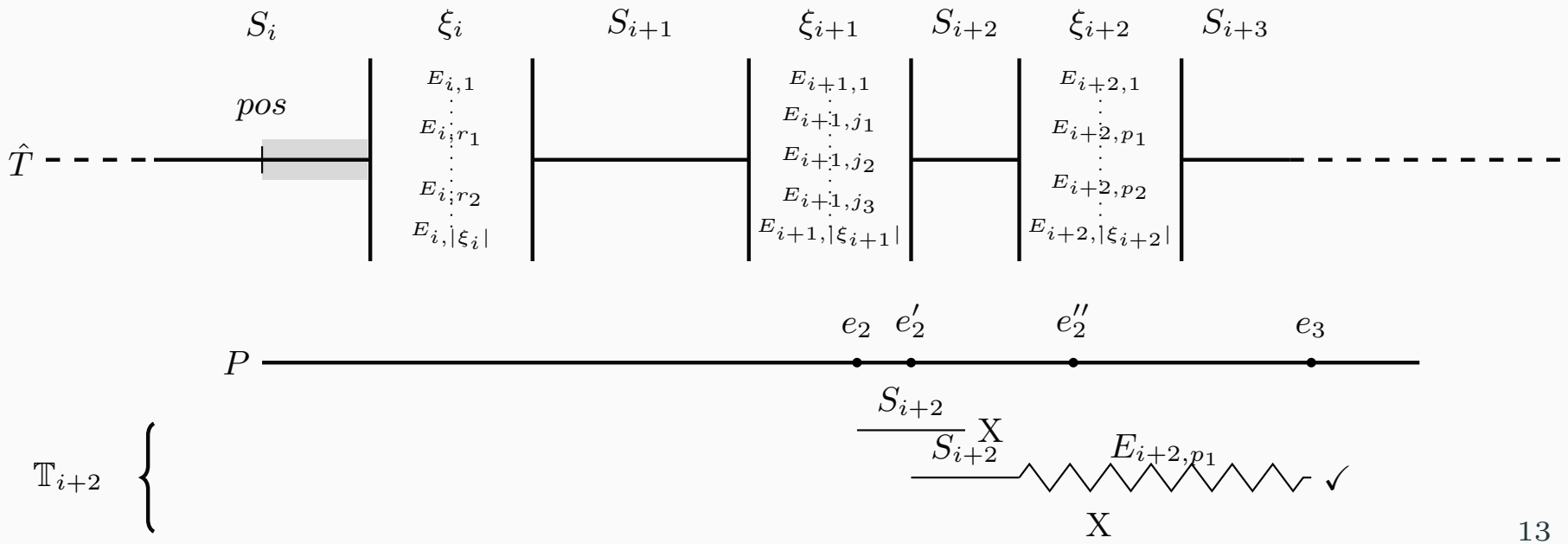

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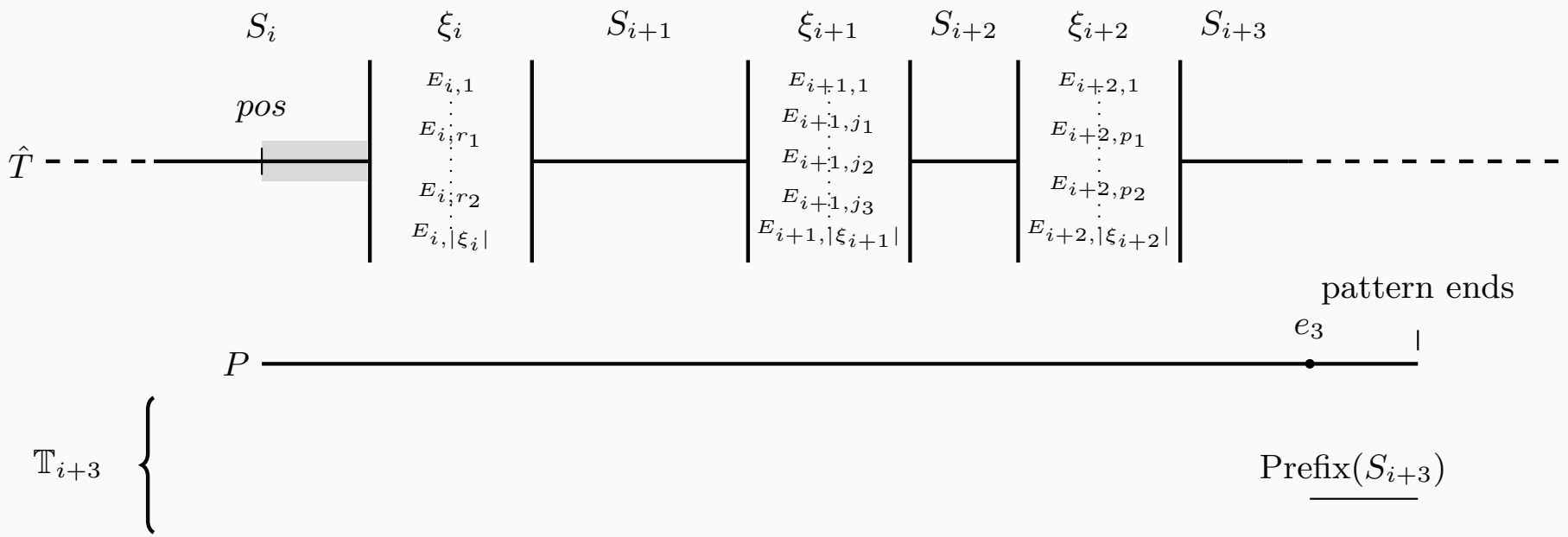

Algorithm

- Preprocessing
- Search
  - Type 1:
  - Type 2:
    - Use KMP to search in as well as marking suffixes of each $E_{i,j}$ appearing as prefixes of P.
    - Incremental extension of marked positions using LCP queries.

ANALYSIS

Analysis

- Preprocessing.
- Search.
  - KMP.
  - Extension:
    - Checking each string in ξ_i: Let parameter α represent the maximum number of strings in any elastic-degenerate symbol of the text.
    - Number of ticks in an iteration.
    - Number of iterations: the number of the ξ_i spanned by the occurrence of P being tested. Let γ is the parameter representing the maximum number of elastic-degenerate symbols spanned by any occurrence of the pattern in the text.

Analysis

- Preprocessing. → $O(N + m)$
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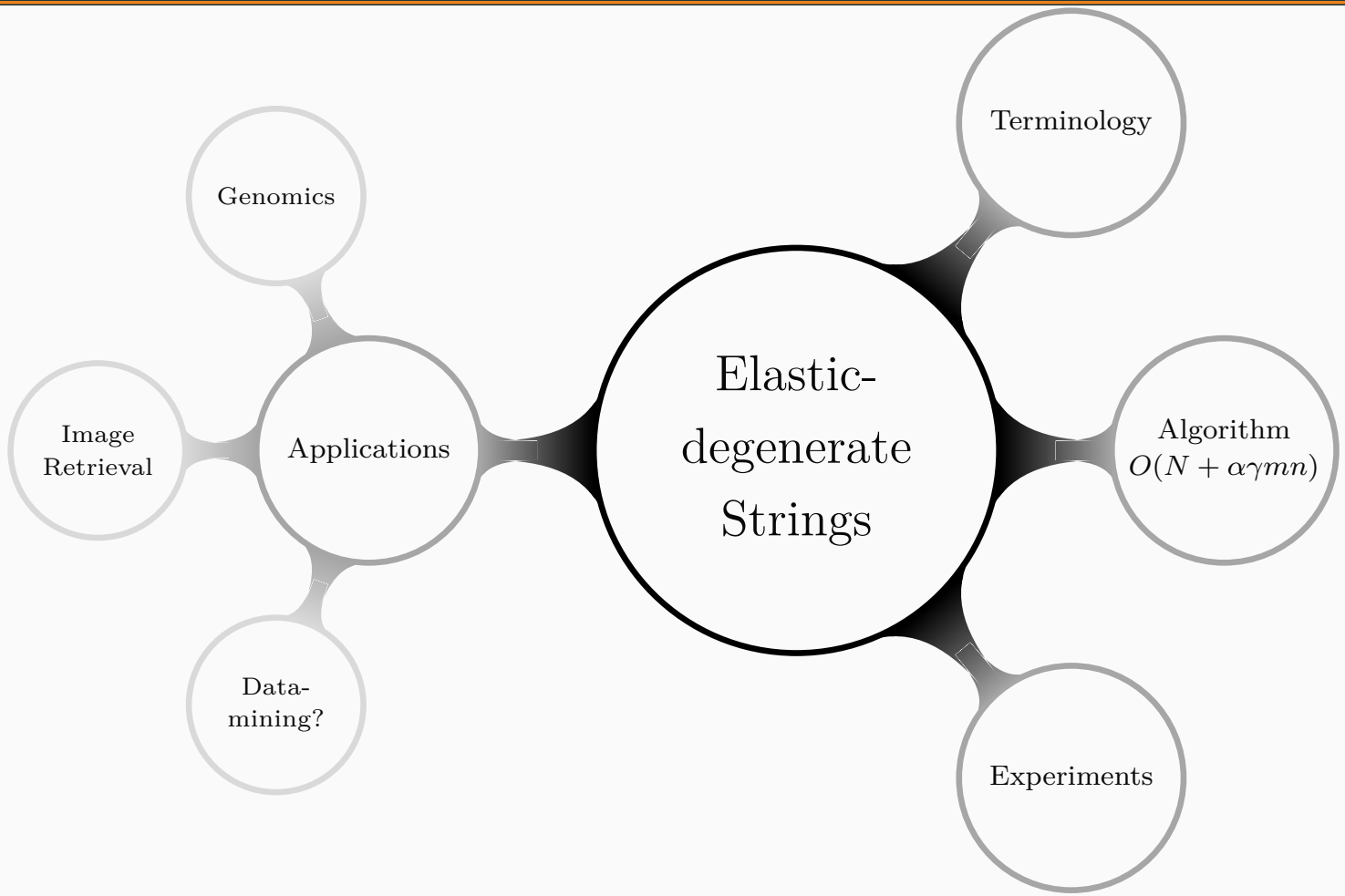
Running time

$O(N + \alpha\gamma mn)$

SUMMARY

Summary



A proof-of-concept implementation of this algorithm can be accessed at https://github.com/Ritu-Kundu/ElDeS.

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Thank You!